EN ESTOS PROBLEMAS SIEMPRE IGUAL
CON F SACAS V.
HACES LA GRAFICA SABIENDO QUE $E_m = E_c + E_p$.

* Ejercicio 17.

$$F = \frac{36}{x^3} - \frac{9}{x^2} \quad (x > 0)$$

$$36 \cdot \frac{1}{x^3} = 36 \cdot x^{-3} = 36\,\frac{x^{-2}}{-2} = -18\,x^{-2} = -\frac{18}{x^2}$$

$$V = -\int \vec{F}\,dx = -\int \frac{36}{x^3} - \frac{9}{x^2}\,dx = -\left(-\frac{18}{x^2} + \frac{9}{x}\right) = +\frac{18}{x^2} - \frac{9}{x} + \cancel{C}^{\,0}$$

Para dibujar la gráfica:

máx y min: $V' = -F = -\frac{36}{x^3} + \frac{9}{x^2} = 0$

$$\frac{-36x^2 + 9x^3}{x^3} = 0 \rightarrow 36x^2 - 9x^3 = 0$$

$$x^2(36 - 9x) = 0$$

$\hookrightarrow 36 - 9x = 0 \Rightarrow x = 4$

$\hookrightarrow \cancel{x = 0}$ el enunciado te dice que $x > 0$

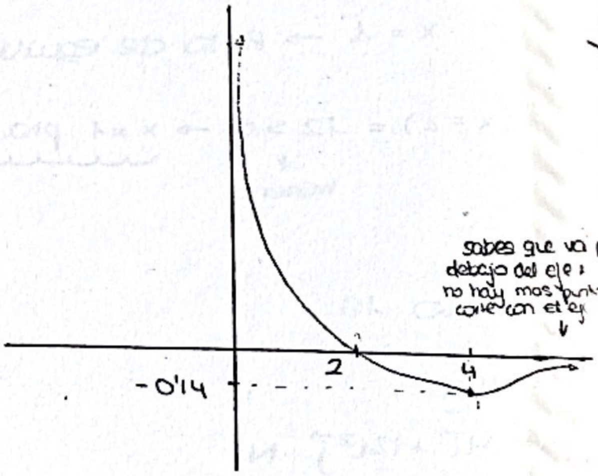
$$V'' = -\frac{108}{x^4} + \frac{18}{x^3} \rightarrow V''(x=4) = -0'14 < 0 \rightarrow \text{mínimo.}$$

corte con el eje x: $V = \frac{18}{x^2} - \frac{9}{x} = 0$

$$18 - 9x = 0 \Rightarrow x = 2.$$

corte con el eje y: igualamos $x = 0$.

$$\frac{18}{0} - \frac{9}{0} = \infty$$



→ cuando la Emec. pasa por encima del eje x, solo hay un punto de corte → mov. libre con un único extremo.

¡OJO! cuando la Emec. pasa por debajo del eje x, hay dos puntos de corte con la gráfica de V →

$x_0 = 4$

$V_0 = 0'5\, m/s$

$m = 1$

$$F = -\frac{dV}{dx} \rightarrow V = -\int F\,dx$$

$$V = -\int \frac{36}{x^3} - \frac{9}{x^2}\,dx = \frac{18}{x^2} - \frac{9}{x}$$

en los extremos $\rightarrow$ velocidad $= 0$

$T = 0 \rightarrow E_m = V$

$$\underbrace{\frac{1}{2} m v_0^2}_{0'125} + \overbrace{\frac{18}{x_0^2} - \frac{9}{x_0}}^{-1'125} = \frac{18}{x^2} - \frac{9}{x} \Rightarrow -1 = \frac{18 - 9x}{x^2}$$

$$-x^2 = 18 - 9x$$

$$x^2 - 9x + 18 = 0$$

$$x = \frac{9 \pm \sqrt{81 - 72}}{2} = \frac{9 \pm 3}{2} = \begin{cases} 6 \\ 3 \end{cases} \text{ Ptos. extremo.}$$

periodo de oscilación. ¡OJO!

$$\tau = 2\int_a^b \frac{dx}{\sqrt{\frac{2(E_0 - V(x))}{m}}} = 2\int_3^6 \frac{1}{\sqrt{\frac{2\left(-1 - \frac{18}{x^2} + \frac{9}{x}\right)}{1}}}\,dx = 2\int_3^6 \frac{1}{\sqrt{-2 - \frac{36}{x^2} + \frac{18}{x}}}\,dx =$$

$$= 2\int_3^6 \frac{1}{\underbrace{\left(-2 - \frac{36}{x^2} + \frac{18}{x}\right)}^{1/2}}\,dx = 2\int_3^6 \frac{1}{\left(\frac{2}{x^2}(x-6)(x-3)\right)^{1/2}}\,dx = 2\int_3^6 \frac{1}{\frac{\sqrt{2}}{x}\left((x-6)(x-3)\right)^{1/2}}\,dx =$$

$$\frac{-2x^2 - 36 + 18x}{x^2} = \frac{2}{x^2}(-x^2 + 18 - 9x) = \frac{2}{x^2}(x-6)(x-3)$$

$$= \frac{2}{\sqrt{2}}\int^6 \frac{x}{\left((x-6)(x-3)\right)^{1/2}}\,dx = \frac{\cancel{2}}{\sqrt{2}}\,\frac{\pi(\overset{3}{a} + \overset{6}{b})}{\cancel{2}} = \frac{9\pi}{\sqrt{2}}$$

$$\frac{1}{2} m v^2 + \frac{18}{x^2} - \frac{9}{x} = -1 \longmapsto \frac{1}{2}\left(\frac{dx}{dt}\right)^2 = -1 + 9/x - 18/x^2$$

$$\dot{x} = \sqrt{\frac{2}{x^2}(x-3)(6-x)} \longrightarrow \frac{dx}{dt} = \pm\sqrt{2}\cdot\frac{\left[(x-3)(6-x)\right]^{1/2}}{x}$$

$$\int_3^6 \frac{x\,dx}{\sqrt{2}\,(x-3)(6-x))^{1/2}} = \int_0^{\tau/2} dt = \tau/2$$

$$\tau = \frac{2}{\sqrt{2}}\int^6 \frac{x\,dx}{(x-3)(6-x)} = \sqrt{2}\,\frac{9\pi}{2} = \frac{9\pi}{\sqrt{2}}\, s$$